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Vortex solutions in nonabelian Higgs theories

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Abstract

A new class of vortex solutions is found in $SU(2)$ gauge theories with two adjoint representation Higgs bosons. Implications of these new solutions and their possible connection with Center Gauge fixed pure gauge theories are discussed.

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The classification of vortex solutions in $SU(N)$ gauge theories requires the investigation of mapping rotations around the vortex axis ($U(1)$ group) into $SU(N)$. Unfortunately, all such maps are continuously deformable into the trivial map. If one considers the group $SU(N)/Z_N$ instead of $SU(N)$ then one finds nontrivial homotopy classes classified by the abelian group, Z_N. Then vortex solutions of finite free energy (but infinite energy) could exist in four-dimensional and finite energy solitons could exist in three-dimensional $SU(N)/Z_N$ gauge theories. [1] A prime candidate for finding such vortex solutions would be an $SU(N)$ gauge theory with adjoint representation Higgs bosons.

Vortex solutions in nonabelian Higgs theories were found some time ago by de Vega and Schaposnik. [2] [3] [4] [5] In view of recent interest in vortex solutions in center gauge fixed nonabelian gauge theories [6][7][8] the existence and properties of vortex solutions is of considerable interest. After the discussion of the new class of vortex solutions we will point out a possible relationship between the two kinds of vortex solutions.

Higgs theories are defined by the Higgs potential. The Higgs potential is not unique for theories with multiple Higgs bosons. The existence of vortex solutions requires the interaction the Higgs bosons with each other. These interactions are designed to keep the Higgs bosons non-parallel at infinite distance away from the vortex line so that they would be able to break the gauge symmetry completely. In previous work a simple mutual interaction, forcing the Higgses to be orthogonal at infinity was used

$$V(\Phi) = \frac{g}{16}[\mathrm{Tr}(\Phi^1\Phi^2)]^2 + \dots, \tag{1}$$

where we omit terms depending on a single Higgs field only. A Higgs theory with such an interaction potential allows for classical solutions with orthogonal Higgs fields. There are vortex solutions in gauge fixed pure lattice gauge theories [8] that, as we will point out later, can be related to a gauge theory with pair of adjoint Higgs fields that

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are *not orthogonal to each other*. Therefore, it is of considerable interest to investigate the existence of vortex solutions in a wider class of Higgs theories that allow non-orthogonal Higgs bosons. In particular we will generalize (2) such that the interaction term has the form

$$V(\Phi) = \frac{g}{4}\left[\frac{1}{2}\mathrm{Tr}(\Phi^1\Phi^2) - c\right]^2 + \ldots \tag{2}$$

where c is the cosine of the "angle" between the two Higgs bosons at infinite distance away from the vortex line. It turns out that such a generalization requires the generalization of the ansatz [2] [3] for the Higgs boson solutions.

Though much of setting up the problem parallels Ref. [2], for completeness, and for establishing notation, we will provide a more or less complete derivation. Though at first we will consider an $SU(N)$ gauge theory, for simplicity, calculations will be restricted to $N = 2$. We hope to return to the case of $N > 2$ in a future publication.

Due to the nontrivial fundamental group of $SU(N)/Z_N$ classical solutions corresponding to gauge transformations that vary smoothly from one element of the center, Z_N, to another one, as one goes around a vortex line on a large circle, are stable. Since the elements of the center are smoothly connected through elements of the Cartan subgroup, $U(1)^{N-1}$, vortex configurations can only appear if the $SU(N)/Z_N$ part of the gauge is fixed completely.

An adjoint Higgs boson is represented by a self adjoint $N \times N$ $SU(N)$ matrix that can always be diagonalized. Gauge transformations that commute with this diagonal matrix form a $U(1)^{N-1}$ Cartan subgroup of the gauge group. In other words, one Higgs boson in the adjoint representation fixes only the gauge group to its Cartan subgroup. Therefore, at least two, non-parallel, adjoint Higgs bosons are required to fix $SU(N)/Z_N$ completely. Accordingly, we set out to search for vortex solutions in a Higgs theory with two adjoint Higgs bosons. To simplify the algebra we will restrict this part of the discussion to $SU(2)$.

As we study time and z coordinate independent solutions our task is to minimize the Hamiltonian,

$$H = \int d^2x \left\{ \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{\Phi_0^2}{2}\sum_{s=1}^{2}[D_\mu\vec{\Phi}^{(s)}]^2 + \frac{\lambda_1\Phi_0^4}{8}\sum_{s=1}^{2}\left(\vec{\Phi}^{(s)2} - 1\right)^2 + \frac{\lambda_2\Phi_0^4}{4}\left(\vec{\Phi}^{(1)}\vec{\Phi}^{(2)} - c\right)^2 \right\}, \tag{3}$$

to find vortex solutions. In (3) c is the "angle" between asymptotic fields, and as such, it should be chosen to be in the interval $-1 \leq c \leq 1$. Φ_0 represents the vacuum expectation value of the Higgs fields. The rescaled, adjoint representation Higgs boson fields, $\vec{\Phi}^{(s)}$, are written in a three-vector form.

The coupling λ_2 is needed to break the $U(1)$ (Cartan) subgroup of $SU(2)$. Notice that at $\lambda_2 = 0$ or at $\lambda_2 \neq 0$ and $c = \pm 1$ this symmetry is not broken as the solution of minimal energy is obtained when $\vec{\Phi}^{(1)} = \pm\vec{\Phi}^{(2)}$ and then gauge rotations around their common direction are symmetries of the system.

By allowing $c \neq 0$ (3) is a generalization of the Hamiltonian of Ref. [2] [3] As we will see later, this generalization leads to Higgs fields that rotate as a function of r. At the same time, to simplify the algebra and concentrate only on the effect of the new constant c, we set the self-coupling and the vacuum expectation value of the two Higgs bosons equal.

The gauge fields are related to the vector potential as

$$\vec{G}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + e\vec{W}_\mu \times \vec{W}_\nu. \tag{4}$$

The covariant derivative of the Higgs fields is defined as

$$D_\mu \vec{\Phi}^{(s)} = \partial_\mu \vec{\Phi}^{(s)} + e\vec{W}_\mu \times \vec{\Phi}^{(s)}. \tag{5}$$

We fix the gauge by setting $\vec{W}_0 = \vec{W}_3 = 0$ and by imposing the gauge condition

$$\partial_\alpha \vec{W}_\alpha = 0, \tag{6}$$

where the subscript α runs over $\alpha = 1, 2$. The general form of the vector potential, satisfying (6) that is finite at the origin is

$$\vec{W}_\alpha = \epsilon_{\alpha\beta}\, x_\beta\, \vec{w}(r). \tag{7}$$

We will discuss boundary conditions at $r = \infty$ later.

Note that though the topological background is different, the form of the vector potential is very similar to the vortex solution in abelian Higgs gauge theory. [9] [10] The vortices of $U(1)$ gauge theory are nothing else but the covariant forms of magnetic flux tubes in a superconductor. Thus, the model we consider represents a non-abelian superconductor.

We will use a singular gauge transformation, $U(\phi)$, in the ansatz for the Higgs bosons. The fundamental group of $SU(2)/Z_2$ is Z_2 so there is only one nontrivial homotopy class that corresponds to gauge orbits that connect two opposite points in the S_4 representation of $SU(2)$. We choose $U(\phi)$ such that it belongs to the nontrivial homotopy class. In other words, we will seek solutions of the field equations of the form

$$\vec{\Phi}^{(s)} \cdot \vec{\sigma} = U(\phi)\, \vec{\psi}^{(s)}(r) \cdot \vec{\sigma}\, U^\dagger(\phi), \tag{8}$$

where ϕ is the angle and r is the radial distance in the xy plane. The singular gauge transformation, $U(\phi)$, is defined as

$$U(\phi) = e^{i\,\phi\,\vec{\sigma}\cdot\hat{a}/2}, \tag{9}$$

where $\hat{a}$ is a constant unit vector. When ϕ runs from 0 to 2π the gauge transformation $U(\phi)$ runs from the identity, I, to the nontrivial element of the center, $-I$. Using gauge transformation we can always choose the unit vector $\hat{a}$ to point in the direction of the positive 3rd axis.

Clearly, the gauge transformation, $U(\phi)$, cannot be deformed continuously to identity. Have we chosen $U(\phi)$ with twice the phase ($\phi \to 2\phi$), this would not be true. Then the transformation would have gone around a large circle of the S_4 sphere when ϕ varies from 0 to 2π, and such a circle could be shrunk to a point through a series of continuous deformations.

The boundary conditions at $r = 0$ require that $\vec{\psi}^{(s)}(0) = \lambda\hat{a}$, otherwise $\Phi^{(s)}$ would be singular at the origin and $\int dr\, r(\partial_\mu \Phi)^2$ would diverge. The finiteness of the self-interaction terms of (3) also requires that $[\vec{\psi}^{(s)}(\infty)]^2 = 1$ and $\vec{\psi}^{(1)}(\infty) \cdot \vec{\psi}^{(2)}(\infty) = c$.

Note that (7) implies that $\vec{W}_\mu \times \vec{W}_\nu = 0$. Then the field equations simplify considerably. The field equation for the gauge field after multiplying with $U(\phi)$ from the

right and $U^\dagger(\phi)$ from the left will contain terms such as $U^\dagger\, \vec{w}\cdot\vec{\sigma}\, U$ and $\vec{w}\cdot\vec{\sigma}$, multiplied by functions dependent on r only. These terms have the same ϕ dependence only if $\vec{w}\cdot\vec{\sigma}, U] = 0$. In other words, $\vec{w}$ must be parallel to $\hat{a}$. We will define the function w by the relation $\vec{w} = w\hat{a}$.

Before we proceed with minimizing the Hamiltonian we must make sure that our ansatz is consistent with the field equations. We have seen already that the ϕ dependence of the field equations cancels with the choice of $\vec{w} = w\hat{a}$. All three of the field equations are vector equations in isotopic space. As the gauge fields themselves are of the form $\vec{w} = w\hat{a}$, the equations are consistent with structure if every term is of this form. Suppressing the argument r we arrive at the condition

$$\sum_s \vec{\psi^{(s)}}' \times \vec{\psi}^{(s)} + ew \sum_s \vec{\psi}^{(s)} \times [\vec{\psi}^{(s)} \times \hat{a}] \sim \hat{a}. \tag{10}$$

The components of the terms on the left hand side of (10), orthogonal to $\hat{a}$ are bilinear in the components of the two Higgs field, but linear in their components parallel to $\hat{a}$ and also linear in their orthogonal components. Non-trivial solutions of (10) are offered by the choices

$$\vec{\psi}_a^{(s)} = \pm\vec{\psi}_a^{(2)}, \;\; \vec{\psi}_\perp^{(1)} = \mp\vec{\psi}_\perp^{(2)}, \tag{11}$$

where $\vec{\psi}_\perp^{(s)}$ and $\vec{\psi}_a^{(s)}$ denotes the components of the Higgs fields orthogonal and parallel to $\hat{a}$, respectively.

The field equations for $\vec{\psi}^{(1)}(r)$ and $\vec{\psi}^{(2)}(r)$ have the form

$$\vec{\psi}^{(1)}{}'' + \frac{\vec{1}}{r}\vec{\psi}^{(1)}{}' - e^2r^2\left(w + \frac{1}{e\,r^2}\right)^2 \vec{\psi}_\perp^{(1)} - \frac{\lambda_1}{2}\Phi_0^2\,\vec{\psi}^{(1)}[\vec{\psi}^{(1)2} - 1] - \frac{\lambda_2}{2}\Phi_0^2\,\vec{\psi}^{(2)}[\vec{\psi}^{(1)}\cdot\vec{\psi}^{(2)} - c] = 0 \tag{12}$$

and a similar equation for $\vec{\psi}^{(2)}(r)$, with $\vec{\psi}^{(1)}(r) \leftrightarrow \vec{\psi}^{(2)}(r)$. Projecting these equation to $\hat{a}$ and to a perpendicular direction we can see that these projections are consistent with (11). Thus, we have proven that ansatz (8) combined with (11) and $\vec{w} = w\hat{a}$ are consistent with the equations of motion.

We are now in the position to be able to write down three scalar equations for w, and the components of the Higgs fields. In view of (11) we use only a total of two components for the two Higgs fields. Note that a gauge rotation allows to rotate $\vec{\psi}_\perp$ into the direction of one of the axes. Therefore, we are able to deal with a single perpendicular component only. Denoting the two independent components of the Higgs fields by ψ_a and $\psi_\perp$ the three field equations are then

$$\frac{3\,w'}{r} + w'' - 2e^2\Phi_0^2\left[w + \frac{1}{er^2}\right]\psi_\perp^2 = 0, \tag{13}$$

$$\psi_\perp'' + \frac{1}{r}\psi_\perp' - e^2r^2\left(w + \frac{1}{e\,r^2}\right)^2\psi_\perp - \frac{\Phi_0^2\,\psi_\perp}{2}\left[\lambda_1(\psi_\perp^2 + \psi_a^2 - 1) + \lambda_2(\pm\psi_a^2 \mp \psi_\perp^2 - c)\right] = 0 \tag{14}$$

and

$$\psi_a'' + \frac{1}{r}\psi_a' - \frac{\Phi_0^2\,\psi_a}{2}\left[\lambda_1(\psi_a^2 + \psi_\perp^2 - 1) - \lambda_2(\pm\psi_a^2 \mp \psi_\perp^2 - c)\right] = 0. \tag{15}$$

(13), (14), and (15) do have nontrivial solutions. As an example, choosing the upper signs,[2] observe that the equations for $\psi_\perp$ and ψ_a decouple at $\lambda_1 = \lambda_2$. (15) becomes independent of $\psi_\perp$ and of w having a constant minimal energy solution, $\psi_a = \sqrt{(1+c)/2}$. Then (14) simplifies to

$$\psi_\perp'' + \frac{1}{r}\psi_\perp' - e^2 r^2 \left(w + \frac{1}{e\,r^2}\right)^2 \psi_\perp - \Phi_0^2 \lambda\, \psi_\perp \left(\psi_\perp^2 - \frac{1-c}{2}\right) = 0. \tag{16}$$

It is easy to recognize the system of equations (13) and (16) as the rescaled version of the equations for the 2+1 dimensional Abelian soliton (relativistic superconductor). [9] [10] If one sets $c = 0$ then it is also equivalent to the equations found in Ref. [2] [3] Numerical investigations of those models show that these equations have a nontrivial solution satisfying the appropriate boundary conditions.

It is easy to visualize the motion of the vectors $\vec{\psi}_1$ and $\vec{\psi}_2$ as r changes between $0 < r < \infty$. At $r = 0$ $\psi_\perp$ vanishes, so the two vectors coincide. They both point into the fixed direction of $\hat{a}$. Then, as r increases, $\vec{\psi}_1$ and $\vec{\psi}_2$ develop opposite components perpendicular to $\hat{a}$, until these components reach the value $\pm\sqrt{(1-c)/2}$ at $r = \infty$. At the same time, the vector potential, being regular at the origin, behaves as $W_\mu = (1/e)\epsilon_{\mu i} x^i / r^2$ at large values of r corresponding to a finite magnetic flux along the z axis, $F = 2\pi/e$. As expected, the vector potential is a pure gauge transformation at infinity, $\delta w_\mu = (-i/e) U^\dagger(\phi)\partial_\mu U(\phi)$.

Solutions of the equation of motion at $\lambda_2 \neq \lambda_1$ also exist. At small $\lambda_1 - \lambda_2$ one can calculate these solutions by perturbation theory. In general, the boundary condition for ψ_z at $r = 0$ is that $\psi_z(0)$=finite. Our investigation of numerical solutions will be presented in a future publication. [11]

One more comment about our solution: If we used different self-coupling for the two Higgs bosons then the solution $\vec{\psi}_{1\perp} = -\vec{\psi}_{2\perp}$ and $\vec{\psi}_1 \cdot \hat{a} = \vec{\psi}_2 \cdot \hat{a}$ would not be admissible. Then one would get separate equations for the four components of the two Higgs fields. The dependence of the Higgs fields on r would become more complicated.

In the remaining part of this letter we would like to point out a possible relationship between the vortices we discussed above with those found using the method of center projection in lattice gauge theories. [6][7][8][12][13] Center projection is a method of realizing the Center Vortex Theory of confinement [14] [15] [16] on lattices. Its aim is to extract the degrees of freedom, most relevant for the nonperturbative properties of nonabelian gauge theories, on a lattice. Center projected theories are theories of interacting vortices. We shall point out that a certain class of center gauge fixing methods may be related to Higgs theories with a pair of adjoint representation Higgs bosons. This relationship offers a possible way to define the so-called thick vortices, [7] the continuum analogue of thin vortices (vortices with a cross section of a single plaquette), appearing after center projection.

Center Vortex Theory as dynamical model for confinement was proposed a long time ago. [14] [15] [16] This picture relies on the condensation and percolation of magnetic vortices labeled by the elements of the center of the gauge group, Z_N. Vortices are one dimensional objects in three dimensional space and two dimensional objects, like strings, in four dimensional spacetime. They can be contrasted to monopoles that

[2]Numerical investigations show that this choice leads to lower energies.

are localized objects in space and one dimensional objects, forming world lines, in four dimensional spacetime. Monopoles are are the fundamental objects in the dual superconductor model of confinement by 't Hooft. [17] It is fairly easy to show that a constant density of percolated and randomly distributed magnetic vortex lines piercing Wilson loops on a lattice results in the area law for Wilson loops and, consequently, leads to confinement.

The realization of these ideas on lattices has been highly successful in $SU(2)$ [6] [7] [12] [8], and very recently, in $SU(3)$.[13] The first step of center projection methods is to fix the gauge to retain only the symmetry corresponding to the center of the gauge group. The gauge fixing is followed by projecting the gauge fields to the center, Z_N, leaving an interactive Z_N gauge theory.

A variety of gauge fixing procedures has been used, with the aim of transforming gauge fields to as close to the center of the group as possible. Among others, the Maximal Center Gauge [6] [12] (MCG) and the Laplacian Center Gauge [8] (LCG) are important to mention. Both methods show convincingly that the resultant Z_2 (or Z_3) gauge theory retains the essential nonperturbative properties of the original nonabelian gauge theory, including confinement (with the correct coefficient in the area law) and chiral symmetry breaking.

The MCG method maximizes the functional

$$S_C = \frac{1}{4}\sum_{\mu,x}\left|\mathrm{Tr}U_\mu^V(x)\right|^2 \tag{17}$$

over gauge transformations, $V(x)$. Here $U_\mu^V(x) = V(x)U_\mu(x)V^\dagger(x+\hat{\mu})$ is the gauge transformed gauge field on the lattice.

It is easy to rewrite (17) in terms of an adjoint representation gauge transformation, which has the form[3]

$$V_{ij}(x) = \frac{1}{2}\mathrm{Tr}[V(x)\sigma_i V^\dagger(x)\sigma_j]$$

and of the $SO(3)$ representation of the gauge fields

$$U_{\mu,ij}(x) = \frac{1}{2}\mathrm{Tr}[U_\mu(x)\sigma_i U_\mu^\dagger(x)\sigma_j]$$

as follows:

$$S_V = \sum_{\mu,x,ijk} V_{ij}(x)U_{\mu,jk}(x)V_{ik}(x+\mu), \tag{18}$$

where the indices run from 1 to 3 in $SU(2)$. Then one needs to maximize (18) over all possible orthogonal matrices $V_{ij}(x)$.

The rows (and columns) of the orthogonal matrix $V_{ij}(x)$ are orthonormal. This constraint is relaxed and the largest eigenvalues and the corresponding eigenvectors, $v_i^{1,2}(x)$, of the laplacian matrix

$$\sum_\mu [U_{\mu,ij}(x)\delta(x-y+\hat{\mu}) + U_{\mu,ji}(x-\hat{\mu})\delta(x-y-\hat{\mu}) - 2\delta_{ij}\delta(x-y)]$$

are found in LCG. [8] After orthonormalizing these vectors at every site one can find the gauge transformation generated by LCG. Note that it is not necessary to find three

[3] Here we use $SU(2)$ notations, though the generalization to $SU(N)$ is straightforward.

orthogonal vectors, as the gauge is completely fixed by two columns of the matrix $V_{ij}(x)$.

Both of these gauge fixing procedures, when using only two columns of the adjoint representation gauge transformation, are equivalent to the minimization of the action of a gauge-Higgs model with two adjoint representation Higgs bosons. Introducing the notation $V_{1i}(x) = \Phi_i^{(1)}(x)$ and $V_{2i}(x) = \Phi_i^{(2)}(x)$ the gauge fixing term becomes the gauge invariant kinetic term of the two Higgs bosons. Adding self and mutual interaction terms one obtains the following Higgs action:

$$S_H[U,\Phi] = \sum_x \left\{ \sum_{r,\mu,ij} \frac{1}{2}\Phi_i^{(r)}(x)U_{\mu,ij}(x)\Phi_j^{(r)}(x+\mu) + \frac{\lambda_1}{8}\sum_{r=1}^{2}[(\vec{\Phi}^{(r)})^2-1]^2 + \frac{\lambda_2}{4}[\vec{\Phi}^{(1)}\cdot\vec{\Phi}^{(2)}-c]^2 \right\}. \tag{19}$$

It should be understood that S_H is used in a way to solve $\partial S_H/\partial\Phi_i^{(r)} = 0$ first, then to define the gauge transformation as the one transforming the minimizing solution, $\Phi_i^{(r)}$, to a pre-determined form, say $\Phi_i^{(1)} + \Phi_i^{(2)} \sim \delta_{i3}$ and $\Phi_i^{(1)} \sim \alpha\delta_{i3} + \beta\delta_{i1}$ or to $\Phi_i^{(1)} \sim \delta_{i3}$ and $\Phi_i^{(2)} \sim \alpha\delta_{i3} + \beta\delta_{i1}$ (this latter prescription was followed by Alexandrou et. al. [8])

It is easy to see that the Higgs gauge fixing term (19) incorporates both MCG and LCG. In the limit $\lambda_1 \to \infty$, $\lambda_2 \to \infty$ and $c \to 0$ the orthonormality of the two Higgs bosons is enforced. In the limit of $\lambda_1 = \lambda_2 = 0$ all these constraints are fully relaxed and the gauge fixing is just like in LCG. Our generalized gauge fixing procedure is then defined by maximizing (19) in the given gauge field background and then choosing the gauge e.g. to rotate one of these Higgs bosons parallel to the z axis and the other one into the xz plane.

The form of (19) is tantalizing, as it offers a possible relationship between gauge theories with two adjoint representation Higgs bosons and Center Gauge fixing. Note that the condition $\partial S_H/\partial\Phi_i^{(r)}$ is identical to the one used in the Higgs theory to find classical vortex solutions. As gauge fixing does not affect the physical gauge fields that were generated without gauge fixing, the extremum condition for the gauge field is not applicable. Still the vortices appearing in the gauge fixed theory satisfy similar boundary conditions at infinity and near the vortex core as in the Higgs theory. Thus, one expects that Center vortices and vortices in the Higgs theory are mathematically very similar. We conjecture that the adjoint Higgs theory is a good laboratory for the analytic investigation of center vortices, mostly for studying interaction and condensation of vortices.

As vortices in center gauge fixed theories condense it would be of considerable interest to investigate multiple vortex configurations in Higgs theories. Note however that e.g. in $SU(2)$ the superposition of two vortices corresponds to the trivial homotopy class and no stable double vortex solutions should exist. [1] In our view this does not mean that such configurations do not give contributions to physical quantities. Indeed, the generating function could be dominated by condensates of vortices, partially due to phase space effects and partially due to the difficulty of annihilating two 'infinitely long' vortices that are not parallel to each other. In a lattice gauge theory the 'speed' of creating these vortices can be in equilibrium with their 'speed' of annihilation.

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